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# Analytical Prediction of Vortex Lift

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**General integral expressions are derived for the nonlinear lift and pitching moment of arbitrary wing planforms in subsonic flow. The analysis uses the suction analogy and an assumed pressure distribution based on classical linear theory results. The potential flow lift constant and certain wing geometric parameters are the only unknowns in the integral expressions. Results of the analysis are compared with experimental data and other numerical methods for several representative wings, including ogee and double-delta planforms. The present method is shown to be as accurate as other numerical schemes for predicting total lift, induced drag, and pitching moment.**

## Nomenclature

| | |
|---|---|
| $\mathcal{R}$ | = aspect ratio |
| $b$ | = wing span |
| $c$ | = chord |
| $\bar{c}$ | = reference length |
| $C_L$ | = lift coefficient |
| $C_D$ | = drag coefficient |
| $C_m$ | = pitching moment coefficient |
| $C_T$ | = thrust coefficient |
| $C_S$ | = suction coefficient |
| $cc_l$ | = section lift coefficient |
| $cc_d$ | = section induced drag coefficient |
| $cc_s$ | = section suction coefficient |
| $\Delta C_p$ | = pressure loading coefficient |
| $D$ | = drag |
| $E_1$ | = proportionality constant, Eq. (32) |
| $E_2$ | = proportionality constant, Eq. (53) |
| $G(\xi)$ | = chordwise function, Eq. (44) |
| $H(\eta)$ | = spanwise function, Eq. (28) |
| $K_p$ | = potential constant |
| $L$ | = lift |
| $P_0$ | = loading constant, Eq. (5) |
| $S$ | = suction force |
| $S_R$ | = reference area |
| $s$ | = suction force per unit length |
| $T$ | = leading edge thrust, Eq. (7) |
| $T'$ | = leading edge thrust per unit length |
| $V$ | = freestream speed |
| $w_i$ | = downwash velocity component, Eq. (11) |
| $\alpha$ | = angle of attack |
| $\Gamma$ | = vorticity |
| $\rho$ | = freestream density |
| $\xi$ | = nondimensional chordwise coordinate |
| $\eta$ | = nondimensional spanwise coordinate |
| $\Lambda$ | = leading edge sweep angle |

*Subscripts*

| | |
|---|---|
| $P$ | = potential flow |
| $E$ | = edge |
| $i$ | = induced |
| $VLE$ | = leading edge vortex |
| $VSE$ | = side edge vortex |

## Introduction

THE nonlinear behavior of lift with angle of attack, which has become known as vortex lift, typically occurs on the thin, highly swept, sharp-edged wings used for transonic and supersonic flight. This phenomenon is most noticeable when the wings are in the subsonic flight regime, although Polhamus[1] also found evidence of vortex lift on highly swept wings in supersonic flow when the leading edge was swept behind the apex Mach wave. The significance of vortex lift is illustrated by the fact that it is currently being incorporated in design features and used advantageously to improve both lift and drag characteristics. Spanwise blowing, for example, augments and controls the leading edge separation vortex and significantly improves lift.[2] Significant drag reduction can also be obtained by cambering sharp leading edges to create suction or thrust.[3]

The general nature of this separated vortex flow is well understood, having been the subject of several enlightening experimental investigations. However, accurate theoretical prediction of these nonlinear effects is still being widely studied, as evidenced by the number of recent journal articles on the subject. Extensive literature surveys and discussions of prediction methods may be found in several references (e.g., Refs. 4 and 5), and will not be repeated here. A comprehensive literature survey is contained in the survey paper by Parker,[5] which includes detailed discussions on the nature of the flow, past and present prediction methods, and the relative degree of success of the theories.

The most successful current methods are numerical, which utilize a linearized potential flow theory lifting surface model. These linear theories alone tend to underpredict lift for sharp-edged wings and must be modified either by use of the suction analogy or by introducing a more complex model of the actual flow.

The leading edge suction analogy, as proposed by Polhamus,[6] lead to accurate theoretical prediction of nonlinear vortex lift on low $\mathcal{R}$ delta wings. The suction analogy is based on the singular nature of the local surface vorticity at leading and side edges of wings with attached potential flow.[7] It was initially verified by comparison of analytical results with experimental data[1] for sharp-edged delta wings; later, the analogy was extended to flat trapezoidal wings with vortex flow along the side edges by Lamar.[8] The vortex lift is accounted for with an additional term in the $C_L$ equation, which can be determined simply from overall potential lift and induced drag. Both lift and drag constants may be computed from any accurate potential flow lifting surface theory.[1] The Polhamus method gives accurate estimates for total loading, but does not give spanwise distributions. Curved leading edges also present a problem because of inaccuracies in predicting local values of the leading edge suction force. This problem was overcome in the approach of Bradley et al.[9]

Except for the Polhamus theory, analytical attempts have generally failed. Most numerical methods use vortex lattice for the lifting surface with discrete, segmented, load-free filaments for the wake. The iterative vortex lattice scheme of Kandil et al.,[4] which models the details of the separation vortex field near the leading edge and wing tips, produces excellent results. The Kandil model, as with previous experimental studies, shows that vortex lift is intimately

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associated with the rollup of vorticity shed from sharp leading and side edges and the influence of this vorticity on the pressure field near these edges. This fact was surmised by Polhamus, and although his suction analogy does not model the actual flowfield, the net effects on lift, drag, and moment are estimated quite well. The Kandil method models leading edge and tip separation and reattachment using a vortex lattice technique which gives distributed loads. The model can handle nonplanar wings in compressible flow.[10]

Unfortunately, such methods are expensive and time consuming for designers who are interested in total $C_L$ and $C_m$ behavior vs $\alpha$, and who desire estimates in an analytical form that can be iterated on and optimized for design purposes.

As noted in Ref. 9, no general empirical or purely analytical methods are available for the prediction of nonlinear lift characteristics of sharp-edged wings, except perhaps for the supersonic method of Ericsson and Reding.[11] The objective of the current paper is to derive accurate, general expressions for vortex lift from a simple analytical model. The model employs an assumed pressure loading distribution based on classical theoretical results and makes use of the suction analogy. The analysis yields a general integral expression for the lift coefficient of any arbitrary planform and reduces to a simple form for trapezoidal planforms. The lift expression contains only a single unknown, the potential constant, and certain wing geometry parameters.

Since an assumed pressure loading distribution is used, the designer is free to specify a desired potential constant, and then solve the relatively simple inverse problem of designing a wing for a specified load distribution. Now, however, the nonlinear lift characteristics of the wing would be known in advance. Expressions for induced drag and pitching moment may be easily obtained by an analysis similar to the one presented for lift. The results of the analysis, applied to several representative wings, are compared with experimental data and with other numerical methods which use the suction analogy.

## Assumed Pressure Loading

Assume that the pressure loading distribution is of the form

$$\Delta C_p = \frac{2P_0}{\pi c}\sqrt{\frac{1-\xi}{\xi}}\sqrt{1-\eta^2}\ \sin\alpha\cos\alpha \tag{1}$$

where $P_0$ is a constant with dimensions of length, and $c$ is the local wing chord. The wing geometry and coordinate system for the terms in Eq. (1) are shown in Fig. 1. The pressure form has been used with great success in a subsonic kernel function method.[12] The proper leading edge singularity and Kutta condition at the trailing edge are satisfied. For wings with finite tip chords, the loading approaches zero at the tips, while for delta wings, classical slender wing behavior is exhibited. The section and total loading coefficients are then

$$cc_l = \cos\alpha\int_{x_{LE}}^{x_{TE}} \Delta C_p \mathrm{d}x = P_0 \sin\alpha\cos^2\alpha\sqrt{1-\eta^2} \tag{2}$$

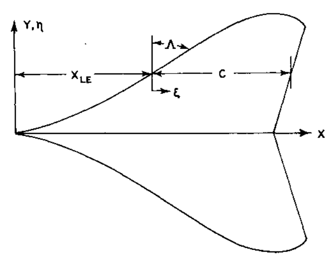


**Fig. 1 Wing geometry and coordinate system.**

$$C_L = \frac{1}{S_R}\int_{-b/2}^{b/2} cc_l \mathrm{d}y = \frac{\pi b P_0}{4S_R}\sin\alpha\cos^2\alpha \tag{3}$$

For purely potential flow, the lift coefficient has been expressed as[6]:

$$C_L = K_p \sin\alpha\cos^2\alpha \tag{4}$$

where $K_p$ is the potential constant. Equating the two lift expressions and solving for $P_0$ gives

$$P_0 = \frac{4S_R K_p}{\pi b} \tag{5}$$

so that

$$\Delta C_p = \frac{8S_R K_p}{\pi^2 bc}\sqrt{\frac{1-\xi}{\xi}}\sqrt{1-\eta^2}\ \sin\alpha\cos\alpha \tag{6}$$

The preceding expression is the one that will subsequently be used to derive general expressions for leading and side edge vortex lift.

## Leading Edge Suction

### Constant Leading Edge Sweep

For the case of a constant leading edge sweep, it is not necessary to employ the assumed pressure loading to calculate the leading edge suction. However, it is necessary to assume that the spanwise distribution of lift is elliptical, which would of course be the case if the assumed pressure loading were realized.

The relation between leading edge thrust, potential flow lift, and potential flow induced drag given by classical finite wing theory is

$$T = L_p \sin\alpha - D_i \cos\alpha \tag{7}$$

Equation (7) is identical to Polhamus's Eq. (7)[6]:

$$T = \rho\Gamma b(V\sin\alpha - w_i) \tag{8}$$

if we interpret the variables as

$$L_p = \rho V \Gamma b \tag{9}$$

and

$$D_i \cos\alpha = \rho\Gamma b w_i \tag{10}$$

According to Polhamus, $w_i$ acts normal to the wing plane and is the component of the net effective downwash due to the trailing vortices. If we assume that the spanwise distribution of lift is elliptic, classical finite wing theory results give

$$w_i = w_0\cos\alpha = \frac{VC_{L_p}}{\pi A\!\!\!R}\cos\alpha \tag{11}$$

Note that since the downwash is constant over the wing, the section induced drag will have an elliptical distribution since

$$cc_{d_i} = \frac{w_0}{V}cc_l \tag{12}$$

This fact will become useful later in computing the suction on wings with curved leading edges. Using Eq. (11) in Eq. (10),

$$D_i\cos\alpha = \rho V\Gamma b\frac{C_{L_p}}{\pi A\!\!\!R}\cos\alpha \tag{13}$$

or

$$C_{D_i} = C_{L_p}^2/\pi A\!\!\!R \tag{14}$$

which is the well-known classical result for elliptic loading. With the results of Eq. (11), Eq. (7) in coefficient form becomes

$$C_T = C_{L_p}\sin\alpha - \frac{C_{L_p}^2}{\pi A\!\!R}\cos\alpha \tag{15}$$

The relation between the thrust and suction coefficients is

$$C_S = C_T/\cos\Lambda \tag{16}$$

where $\Lambda$ is the leading edge sweep angle. Rotating $C_S$ into the normal force plane in accordance with the suction analogy, the expression for the leading edge vortex lift becomes

$$C_{L_{VLE}} = C_S\cos\alpha = \left(C_{L_p}\sin\alpha - \frac{C_{L_p}^2}{\pi A\!\!R}\cos\alpha\right)\frac{\cos\alpha}{\cos\Lambda} \tag{17}$$

The potential flow lift coefficient form, Eq. (4), when used in Eq. (17), results in

$$C_{L_{VLE}} = \left(K_p\sin^2\alpha\cos^2\alpha - \frac{K_p^2}{\pi A\!\!R}\sin^2\alpha\cos^5\alpha\right)\frac{\cos\alpha}{\cos\Lambda} \tag{18}$$

The net lift on the wing is then

$$C_L = C_{L_p} + C_{L_{VLE}} \tag{19}$$

In the Polhamus vortex lift expression,

$$C_L = C_{L_p} + K_v\sin^2\alpha\cos\alpha \tag{20}$$

where

$$K_v = (K_p - K_p^2 K_i)/\cos\Lambda \tag{21}$$

and $K_i$ is a downwash factor which must be determined via "an appropriate lifting surface theory." The present derivation implies

$$K_v = \left(K_p\cos^2\alpha - \frac{K_p^2}{\pi A\!\!R}\cos^5\alpha\right)\Big/\cos\Lambda \tag{22}$$

where only the potential constant $K_p$ need be found. Notice that for moderate angles of attack, Eq. (22) is nearly identical to Eq. (21). However, the accuracy of Eq. (22) remains to be shown. Results of the present derivation, Eqs. (18) and (19), are compared with experimental data and Polhamus' original calculations for several delta wings in Fig. 2. To minimize differences, the values of $K_p$ used in Eqs. (18) and (19) were those originally given by Polhamus (Fig. 3 in Ref. 6).

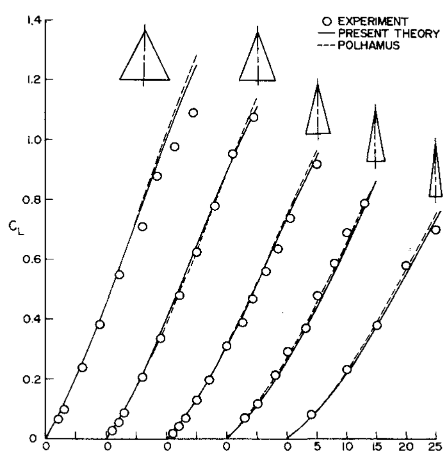


**Fig. 2 Delta wing lift.**

The central assumption of the present method, i.e., that the span loading is elliptic, does not seem too significant in lieu of the results obtained for delta wings. This is not too surprising since Prandtl's original finite wing results predict that additional terms in the lift series do not change the net lift but merely alter the elliptic spanwise distribution.

### Variable Leading Edge Sweep

For wings with variable leading edge sweep, the calculation of the total vortex lift requires a knowledge of the spanwise distribution of vortex induced loading. It would seem logical to assume that the distribution of leading edge thrust is governed by an equation analogous to Eq. (7), such as

$$T' = L_p'\sin\alpha - D_i'\cos\alpha \tag{23}$$

If the pressure loading which generates the thrust is similar to Eq. (1), then the induced downwash will be a constant, and $T'$, $L_p'$, and $D_i'$ will have elliptic distributions. Using the suction analogy and the preceding assumptions, the local suction coefficient is simply $T'/\cos\Lambda$. Rotating the local suction coefficient so that it acts normal to the wing plane then gives a modified but basically elliptic distribution of vortex lift. However, in separated flow, the vortex lift is generated by a separation vortex which rolls up and grows along the span. This implies that the section distribution of suction and thrust should be zero near the wing apex, rather than having a maximum value there as predicted by an elliptic form.

A functional form which has the proper shape may be obtained by assuming that the spanwise rate at which vorticity is shed from the leading edge is proportional to the right-hand side of Eq. (23). The spanwise rate of change of thrust and suction in separated flow may then be written as

$$\frac{d}{dy}T' = \frac{d}{dy}s\cos\Lambda = E_0(L_p'\sin\alpha - D_i'\cos\alpha) \tag{24}$$

where $E_0$ is a constant of proportionality. In coefficient form, the suction is then

$$cc_s = \frac{b}{2}E_0\int_0^\eta (cc_l\sin\alpha - cc_d\cos\alpha)\,d\eta_0 \tag{25}$$

where the integrand is effectively the vorticity shed per unit length from the leading edge. Substitution from Eqs. (2) and (12) for $cc_l$ and $cc_d$, respectively, gives

$$cc_s = \frac{bE_0P_0\sin\alpha\cos^2\alpha(\sin\alpha - \frac{w_i}{V}\cos\alpha)}{2\cos\Lambda}\int_0^\eta \sqrt{1-\eta_0^2}\,d\eta_0 \tag{26}$$

Performing the integration, and using values from Eqs. (4), (5), (11), and (15) results in

$$cc_s = \frac{S_R E_0 C_T}{\pi\cos\Lambda}H(\eta) \tag{27}$$

where $C_T$ is given by Eq. (15) and

$$H(\eta) = \int_0^\eta \sqrt{1-\eta_0^2}\,d\eta_0 = \eta\sqrt{1-\eta^2} + \arcsin\eta \tag{28}$$

The total suction is then

$$C_S = \frac{2}{S_R}\int_0^{b/2} cc_s\,d_y \tag{29}$$

or

$$C_S = \frac{bE_0C_T}{\pi}\int_0^1 \frac{H(\eta)\,d\eta}{\cos\Lambda} \tag{30}$$

If $\Lambda$ is a constant, as in the case of delta or trapezoidal wings, the cosine term may be removed from under the integral. Evaluation of the integral should then result in Eq. (16). In this case, integration and substitution from Eq. (16) gives for $E_0$

$$E_0 = \frac{\pi E_1}{b} \tag{31}$$

where

$$E_1 = \frac{1}{\frac{\pi}{2}-\frac{1}{3}} \cong 1.106036 \tag{32}$$

From the suction analogy, the vortex lift is just $C_S\cos\alpha$. This gives

$$cc_{l_{VLE}} = \frac{S_R E_1 C_k}{b\cos\Lambda} H(\eta) \tag{33}$$

and

$$C_{L_{VLE}} = E_1 C_k \int_0^1 \frac{H(\eta)\,d\eta}{\cos\Lambda} \tag{34}$$

for the section and total vortex lift, respectively, where

$$C_k = C_T\cos\alpha = K_p\sin^2\alpha\cos^3\alpha\left(1-\frac{K_p}{\pi A\!\!\!R}\cos^3\alpha\right) \tag{35}$$

from Eqs. (4) and (15). For double-delta or cranked wings, in which the sweep changes from $\Lambda_1$ to $\Lambda_2$ at a fixed spanwise station $\eta_0$, evaluation of the integral in Eq. (34) leads to

$$C_{L_{VLE}} = E_1 C_k\left[\frac{(\cos\theta_0+\theta_0\sin\theta_0-1/3\cos^3\theta_0-2/3)}{\cos\Lambda_1} + \frac{(\pi/2-\cos\theta_0-\theta_0\sin\theta_0+1/3\cos^3\theta_0)}{\cos\Lambda_2}\right] \tag{36}$$

where $\theta_0=\arcsin\eta_0$. Results of the preceding expression and Eq. (34) are compared in Fig. 3 with numerical results given by Polhamus[1] for two double-delta wings and an ogee wing. The values of $K_p$ used in the present theory were computed using the method from Ref. 12, which did not vary significantly from the values given by Polhamus.

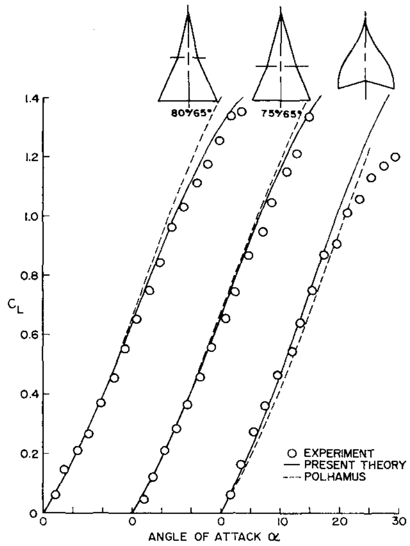

**Fig. 3 Variable sweep wing lift.**

## Side Edge Suction

Lamar[8] has shown that it is possible to account for side edge (wing tip) vortex lift with the suction analogy. We will next attempt to derive an expression for side edge vortex lift based on the assumed pressure loading distribution. The side edge suction per unit length, based on the concepts in Ref. 13, is

$$s(x) = \pi\rho \lim_{\eta\to 1}\frac{b}{2}(1-\eta)v\delta \tag{37}$$

where $b/2(1-\eta)$ is the distance from the wing tip, $v$ is the local spanwise velocity, and $\delta$ is the shed vorticity

$$\delta = \frac{2}{b}\frac{\partial\Gamma}{\partial\eta} \tag{38}$$

The relation between the local spanwise velocity and the shed vorticity is found from the Kutta condition

$$v = \frac{1}{2}\delta \tag{39}$$

The expression for $\Gamma$ is

$$\Gamma = \int_{x_{LE}}^{x} \gamma(x_0,y)\,dx_0 \tag{40}$$

where $\gamma$ is the local bound surface vorticity and is related to the pressure loading by

$$\gamma = \frac{V}{2}\Delta C_p \tag{41}$$

Using the previously assumed pressure loading,

$$\gamma = \frac{VP_0}{\pi c}\sqrt{\frac{1-\xi}{\xi}}\sqrt{1-\eta^2}\,\sin\alpha\cos\alpha \tag{42}$$

and

$$\Gamma = c\int_0^{\xi}\gamma\,d\xi_0 = \frac{VP_0}{\pi}\sqrt{1-\eta^2}\sin\alpha\cos\alpha G(\xi) \tag{43}$$

where

$$G(\xi) = \int_0^{\xi}\sqrt{\frac{1-\xi_0}{\xi_0}}\,d\xi_0 = \sqrt{\xi(1-\xi)} + \arcsin\sqrt{\xi} \tag{44}$$

The chordwise vorticity $\delta$ is then

$$\delta = \frac{2VP_0}{\pi b}\left(\frac{-\eta}{\sqrt{1-\eta^2}}\right)\sin\alpha\cos\alpha G(\xi) \tag{45}$$

The expression for $s(x)$ becomes

$$s(x) = \frac{\pi\rho b}{8}\left(\frac{2VP_0}{\pi b}\right)^2\sin^2\alpha\cos^2\alpha G^2(\xi) \tag{46}$$

The total suction along one tip is

$$S_E=\int_{x_{LE}}^{x_{TE}} s(x)\,\mathrm{d}x=c_t\int_0^1 s(\xi)\,\mathrm{d}\xi \tag{47}$$

or

$$S_E=\frac{\pi\rho b c_t}{8}\left(\frac{2VP_0}{\pi b}\right)^2\sin^2\alpha\cos^2\alpha\int_0^1[\xi(1-\xi)$$

$$+2\sqrt{\xi(1-\xi)}\arcsin\sqrt{\xi}+\arcsin^2\sqrt{\xi}]\,\mathrm{d}\xi \tag{48}$$

Integration of the three terms in brackets gives

$$S_E=\frac{\rho V^2}{2}\left(\frac{c_t}{b}\right)\frac{P_0^2}{\pi}\sin^2\alpha\cos^2\alpha\left(\frac{3\pi^2}{16}-\frac{1}{3}\right) \tag{49}$$

Squaring the previously determined expression for $P_0$ gives

$$S_E=\left(\frac{\rho V^2 S_R}{2}\right)\left(\frac{c_t}{b}\right)\frac{16K_p^2}{\pi^3 A\!\!R}\sin^2\alpha\cos^2\alpha\left(\frac{3\pi^2}{16}-\frac{1}{3}\right) \tag{50}$$

In coefficient form,

$$C_{S_E}=\frac{16}{\pi^2}\left(\frac{3\pi^2}{16}-\frac{1}{3}\right)\left(\frac{c_t}{b}\right)\frac{K_p^2}{\pi A\!\!R}\sin^2\alpha\cos^2\alpha \tag{51}$$

Including both tips, the resultant lift contribution after rotation is $2S_E\cos\alpha$, or

$$C_{L_{VSE}}=E_2\left(\frac{c_t}{b}\right)\frac{K_p^2}{\pi A\!\!R}\sin^2\alpha\cos^3\alpha \tag{52}$$

where

$$E_2=\frac{32}{\pi^2}\left(\frac{3\pi^2}{16}-\frac{1}{3}\right)\cong 4.91924 \tag{53}$$

The similarity between expression (52) and the classical induced drag equation

$$C_{D_i}=C_L^2/\pi A\!\!R \tag{54}$$

is interesting, as is the appearance of the tip chord to span ratio $c_t/b$. Including this term in the expression for total lift gives

$$C_L=C_{L_P}+C_{L_{VLE}}+C_{L_{VSE}} \tag{55}$$

Results are shown in Figs. 4 and 5 for several representative wings.

## Pitching Moment and Induced Drag

Since the distribution of vortex lift along the leading and side edges is available in analytical form, the calculation of the moment induced by these loads is straightforward.

The total pitching moment is

$$C_m=C_{m_p}+C_{m_{VLE}}+C_{m_{VSE}} \tag{56}$$

The pitching moment due to the attached potential flow $C_{m_p}$ can be found using a variety of methods, including integration of the assumed pressure distribution equation. The assumed pressure form yields fairly accurate centers-of-pressure for swept wings; however, for rectangular wings of zero sweep, the formula always gives the center-of-pressure as the quarter chord. The error becomes large for low aspect ratio wings since the actual center-of-pressure moves toward the leading edge as the aspect ratio tends to zero. This problem also appears in calculating the moment due to side edge suction. The most accurate way to determine $C_{m_p}$ is to use a lifting surface method.

The calculation of pitching moment due to vortex lift is a straightforward integration. Using the wing apex as the moment reference point,

$$C_{m_{VLE}}=-\frac{2}{S_R\bar{c}}\int_0^{b/2}x_{LE}\,cc_s(\eta)\,\mathrm{d}y \tag{57}$$

and

$$C_{m_{VSE}}=-\frac{2}{qS_R\bar{c}}\int_{x_{LE_t}}^{x_{TE_t}}x\cdot s(x)\,\mathrm{d}x \tag{58}$$

where $cc_s(\eta)$ and $s(x)$ are given by Eqs. (27) and (46),

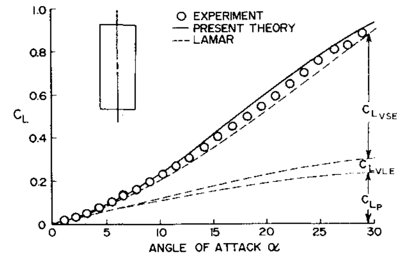


**Fig. 4 Rectangular wing lift.**

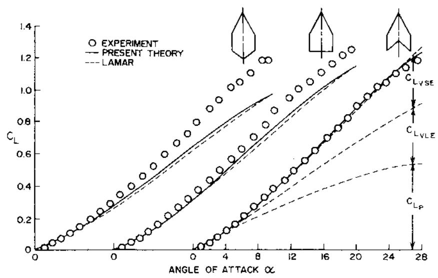


**Fig. 5 Trapezoidal wing lift.**

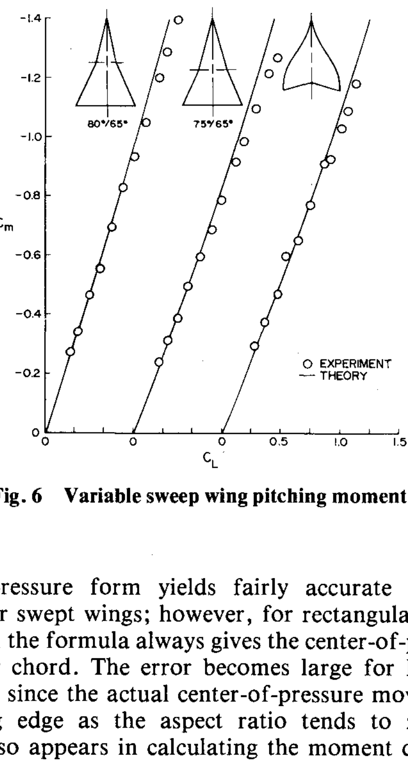


**Fig. 6 Variable sweep wing pitching moment.**

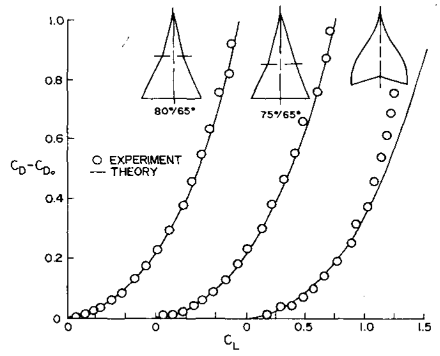


**Fig. 7 Variable sweep wing induced drag.**

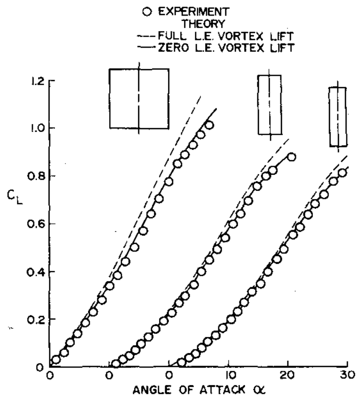


**Fig. 8 Leading edge vortex effect on rectangular wing lift.**

respectively. The integral in Eq. (57) is complicated due to the fact that $x_{LE}$ and $\Lambda$ may both be functions of span. Some typical results are shown in Fig. 6 for an ogee and two double-delta wings. The values of $C_{m_p}$ were obtained using the lifting surface method of Ref. 12. It should be noted that Eq. (58) overpredicts the moment for low aspect ratio rectangular wings.

The effects of vortex lift on drag due to lift is given very accurately by

$$C_D - C_{D_0} = C_L \tan\alpha \tag{59}$$

for most wings, as shown in Fig. 7. There is experimental evidence that the leading edge vortex lift is not developed when the sweep angle approaches zero. Some examples are shown in Fig. 8, where excellent agreement with experiment is obtained when the leading vortex lift is assumed to be zero. Similar results were obtained for total drag in Ref. 8. For zero vortex lift along the leading edge, the drag is

$$C_D - C_{D_0} = C_{D_i} + C_{L_{VSE}} \tan\alpha \tag{60}$$

where $C_{D_i}$ is the usual potential flow induced drag. In the present analysis, no attempt was made to determine the requirements for development of full vortex lift along the leading or side edges.

## Summary

General integral expressions have been derived for analytically predicting the nonlinear lift and pitching moment of arbitrary wing planforms in subsonic flow. The analysis used the suction analogy and an assumed pressure distribution based on classical linear theory results. The potential flow lift constant and certain wing geometric parameters are the only unknowns in the integral expressions, which makes the formulation ideal for use in iterative design procedures. For trapezoidal or delta wings, evaluation of the integrals results in simple algebraic equations. Total lift, induced drag, and pitching moment results for several representative wings are compared with experimental data and other numerical methods.

Excellent correlation with experiment was obtained on most general wing types, including ogee and double-delta. The deficiency in the analysis appears to be in the prediction of the distribution of side edge suction loading, which affects the pitching moment of low aspect ratio rectangular wings. The problems of vortex bursting and onset of vortex lift as a function of leading edge sweep were not addressed.

## Acknowledgment

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